
VIII. Mathematics, Grade 8

Mathematics, Grade 8

The spring 2002 Grade 8 MCAS Mathematics test was based on the learning standards of the Massachusetts *Mathematics Curriculum Framework* (2000). The *Framework* identifies five major content strands:

- Number Sense and Operations
- Patterns, Relations, and Algebra
- Geometry
- Measurement
- Data Analysis, Statistics, and Probability

Curriculum Framework *Learning Standards* for Grades 7-8

Learning standards are grouped below by content strand and are directly quoted from the *Framework*; applicable *Framework* page numbers are shown in parentheses.

Number Sense and Operations (*Framework*, p. 62)

Students engage in problem solving, communicating, reasoning, connecting, and representing as they:

- Compare, order, estimate, and translate among integers, fractions and mixed numbers (e.g., rational numbers), decimals, and percents.
- Define, compare, order, and apply frequently used irrational numbers, such as $\sqrt{2}$ and π .
- Use ratios and proportions in the solution of problems; in particular, problems involving unit rates, scale factors, and rate of change.
- Represent numbers in scientific notation, and use them in calculations and problem situations.
- Apply number theory concepts, including prime factorization and relative prime numbers, to the solution of problems.
- Apply the rules of powers and roots to the solution of problems. Extend the Order of Operations to include positive integer exponents and square roots.
- Demonstrate an understanding of the properties of arithmetic operations on rational numbers. Use the associative, commutative, and distributive properties; properties of the identity and inverse elements (e.g., $-7 + 7 = 0$; $3/4 \times 4/3 = 1$); and the notion of closure of a subset of the rational numbers under an operation (e.g., the set of odd integers is closed under multiplication but not under addition).

- Use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems (e.g., multiplying by $\frac{1}{2}$ or 0.5 is the same as dividing by 2).
- Estimate and compute with fractions (including simplification of fractions), integers, decimals, and percents (including those greater than 100 and less than 1).
- Determine when an estimate rather than an exact answer is appropriate and apply in problem situations.
- Select and use appropriate operations—addition, subtraction, multiplication, division, and positive integer exponents—to solve problems with rational numbers (including negatives).

Patterns, Relations, and Algebra (*Framework*, p. 63)

Students engage in problem solving, communicating, reasoning, connecting, and representing as they:

- Extend, represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic expressions. Include arithmetic and geometric progressions (e.g., compounding).
- Evaluate simple algebraic expressions for given variable values (e.g., $3a^2 - b$ for $a = 3$ and $b = 7$).
- Demonstrate an understanding of the identity $(-x)(-y) = xy$. Use this identity to simplify algebraic expressions (e.g., $(-2)(-x + 2) = 2x - 4$).
- Create and use symbolic expressions and relate them to verbal, tabular, and graphical representations.
- Identify the slope of a line as a measure of its steepness and as a constant rate of change from its table of values, equation, or graph. Apply the concept of slope to the solution of problems.
- Identify the roles of variables with an equation (e.g., $y = mx + b$) expressing y as a function of x with parameters m and b .
- Set up and solve linear equations and inequalities with one or two variables, using algebraic methods, models, and/or graphs.
- Explain and analyze—both quantitatively and qualitatively, using pictures, graphs, charts, or equations—how a change in one variable results in a change in another variable in functional relationships (e.g., $C = \pi d$, $A = \pi r^2$ (A as a function of r), $A_{\text{rectangle}} = lw$ ($A_{\text{rectangle}}$ as a function of l and w)).
- Use linear equations to model and analyze problems involving proportional relationships. Use technology as appropriate.
- Use tables and graphs to represent and compare linear growth patterns. In particular, compare rates of change and x - and y -intercepts of different linear patterns.

Geometry (*Framework, p. 64*)

Students engage in problem solving, communicating, reasoning, connecting, and representing as they:

- Analyze, apply, and explain the relationship between the number of sides and the sums of the interior and exterior angle measures of polygons.
- Classify figures in terms of congruence and similarity, and apply these relationships to the solution of problems.
- Demonstrate an understanding of the relationships of angles formed by intersecting lines, including parallel lines cut by a transversal.
- Demonstrate an understanding of the Pythagorean theorem. Apply the theorem to the solution of problems.
- Use a straightedge, compass, or other tools to formulate and test conjectures, and to draw geometric figures.
- Predict the results of transformations on unmarked or coordinate planes and draw the transformed figure (e.g., predict how tessellations transform under translations, reflections, and rotations).
- Identify three-dimensional figures (e.g., prisms, pyramids) by their physical appearance, distinguishing attributes, and spatial relationships such as parallel faces.
- Recognize and draw two-dimensional representations of three-dimensional objects (e.g., nets, projections, and perspective drawings).

Measurement (*Framework, p. 65*)

Students engage in problem solving, communicating, reasoning, connecting, and representing as they:

- Select, convert (within the same system of measurement), and use appropriate units of measurement or scale.
- Given the formulas, convert from one system of measurement to another. Use technology as appropriate.
- Demonstrate an understanding of the concepts and apply formulas and procedures for determining measures, including those of area and perimeter/circumference of parallelograms, trapezoids, and circles. Use technology as appropriate.
- Use ratio and proportion (including scale factors) in the solution of problems, including problems involving similar plane figures and indirect measurement.
- Use models, graphs, and formulas to solve simple problems involving rates (e.g., velocity and density).

Data Analysis, Statistics, and Probability (*Framework*, p. 66)

Students engage in problem solving, communicating, reasoning, connecting, and representing as they:

- Describe the characteristics and limitations of a data sample. Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling).
- Select, create, interpret, and utilize various tabular and graphical representations of data, (e.g., circle graphs, histograms, tables, and charts).
- Find, describe, and interpret appropriate measures of central tendency (mean, median, and mode) and spread (range) that represent a set of data. Use these notions to compare different sets of data.
- Use tree diagrams, tables, organized lists, basic combinatorics (“fundamental counting principle”), and area models to compute probabilities for simple compound events (e.g., multiple coin tosses or rolls of dice).

MCAS Reporting Categories

In *Test Item Analysis Reports* and on the *Subject Area Subscore* pages of the MCAS *School* and *District Reports*, Mathematics test results are reported under the following five MCAS reporting categories, which are identical to the five *Mathematics Curriculum Framework* content strands:

- Number Sense and Operations
- Patterns, Relations, and Algebra
- Geometry
- Measurement
- Data Analysis, Statistics, and Probability

MCAS Spring 2002 Common Test Items

Mathematics, Grade 8

Test Sessions

MCAS grade 8 Mathematics Student Test Booklets contained 2 separate test sessions. Each session included multiple-choice and open-response questions. Session 1 also included short-answer questions.

Reference Materials and Tools

During testing, each grade 8 student was provided with a **Mathematics Reference Sheet**. A sample of the Grade 8 Mathematics Reference Sheet is included in Appendix A of this document.

During Session 2, each grade 8 student was allowed to use a personal calculator while answering test questions. If any student could not provide his or her own calculator with at least four functions and a square root key, one was provided to that student for use during Session 2. Calculator use was not allowed during Session 1.

No other reference tools or materials were allowed during any grade 8 Mathematics test session.

Cross-Reference Information

The shaded bar underneath each item indicates the item's MCAS reporting category, which is also the name of the *Framework* content strand that contains the learning standards assessed by the item.

Mathematics, Grade 8

Session 1, Multiple-Choice Questions



- 1 The square root of 31 is between which two whole numbers?
- A. 4 and 5
 - B. 5 and 6
 - C. 6 and 7
 - D. 7 and 8

Reporting Category for Item 1: Number Sense and Operations

- 2 Find the next two terms in the sequence shown below.

1, 3, 7, 15, 31, ?, ?

- A. 53, 117
- B. 63, 127
- C. 73, 137
- D. 83, 147

Reporting Category for Item 2: Patterns, Relations, and Algebra

- 3 The regular price of a CD player is \$74. It is on sale for 20% off. Which of the following is **closest** to the sale price?
- A. \$40
 - B. \$50
 - C. \$60
 - D. \$70

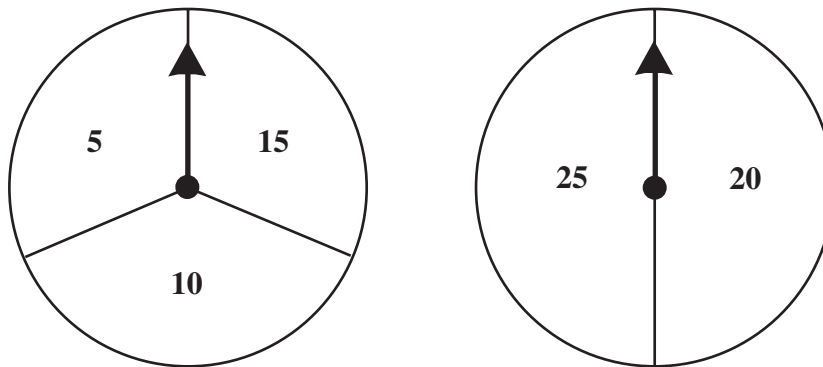
Reporting Category for Item 3: Number Sense and Operations

Mathematics, Grade 8

- 4 For which of the following would a yard be the **most** appropriate unit of measure?
- A. the area of a street sign
 - B. the volume of a bus
 - C. the length of a cafeteria table
 - D. the distance from Boston to Springfield, Massachusetts

Reporting Category for Item 4: Measurement

- 5 May Ling has two spinners, as shown below. She will spin each arrow once and add the results.



Which of the following sets represents every possible outcome?

- A. {25, 30, 35, 40}
- B. {15, 20, 25, 30, 35, 40}
- C. {20, 25, 30, 35, 40, 45}
- D. {10, 15, 20, 25, 30, 35, 40, 45, 50}

Reporting Category for Item 5: Data Analysis, Statistics, and Probability

Mathematics, Grade 8

6 The rates that Zack charges for baby-sitting are as follows:

- \$2.50 per hour for the first child and
- an extra 75¢ per hour for each additional child.

Which chart below shows Zack's hourly charges for baby-sitting one, two, and three children?

A.

Number of children	Hourly charges
1	\$2.50
2	\$3.25
3	\$3.25

B.

Number of children	Hourly charges
1	\$3.25
2	\$4.00
3	\$4.75

C.

Number of children	Hourly charges
1	\$2.50
2	\$3.25
3	\$4.00

D.

Number of children	Hourly charges
1	\$3.25
2	\$4.00
3	\$4.00

Reporting Category for Item 6: *Patterns, Relations, and Algebra*

Mathematics, Grade 8

Session 1, Short-Answer Questions



- 7 Natalie bought a book that was on sale for 25% off. The regular price of the book was \$18. What was the sale price of the book?

Reporting Category for Item 7: Number Sense and Operations

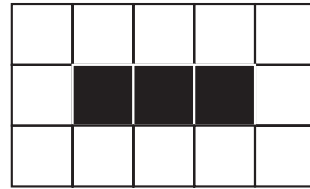
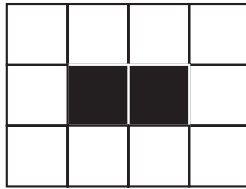
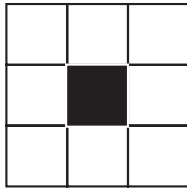
- 8 There are yellow, pink, and purple balloons in a package. If Benjamin takes 1 balloon without looking, the probability that it will be yellow is $\frac{1}{2}$. The probability that it will be pink is $\frac{1}{3}$. The probability that it will be purple is $\frac{1}{6}$. What is the **least** number of balloons that could be in the package?

Reporting Category for Item 8: Data Analysis, Statistics, and Probability

Session 1, Open-Response Question



- 9 A worker placed white tiles around black tiles in the pattern shown in the three figures below.



- Based on this pattern, how many white tiles would be needed for 4 black tiles?
- Based on this pattern, how many white tiles would be needed for 50 black tiles?
- Make a scatterplot of the first five figures in this pattern showing the relationship between the number of white tiles and the number of black tiles. Be sure to label the axes.
- Based on this pattern, explain how you could find the number of white tiles needed for any number, n , of black tiles. Show or explain your work.

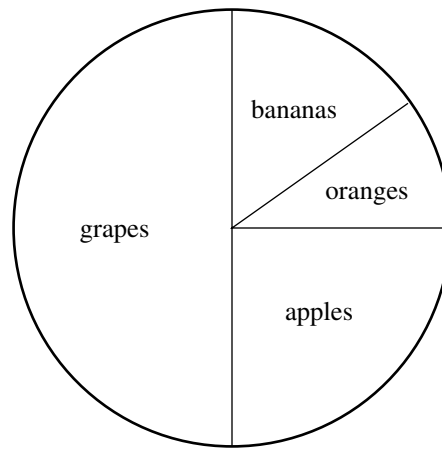
Reporting Category for Item 9: *Patterns, Relations, and Algebra*

Session 1, Multiple-Choice Questions



- 10 Marty surveyed 24 students and asked them to name their favorite fruit. The circle graph below shows the results of his survey.

Students' Favorite Fruits



Which fruit was the favorite of **exactly** 6 of the students?

- A. apples
- B. oranges
- C. bananas
- D. grapes

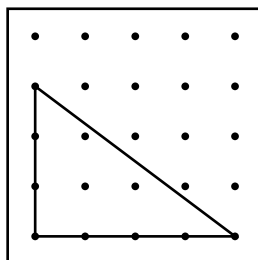
Reporting Category for Item 10: *Data Analysis, Statistics, and Probability*

Mathematics, Grade 8

- 11 Huey is reading a book that is 697 pages long. He tells a friend that he is about $\frac{3}{4}$ of the way done. About how many more pages must Huey read before he finishes the book?
- A. 150 pages
 - B. 160 pages
 - C. 175 pages
 - D. 250 pages

Reporting Category for Item 11: *Number Sense and Operations*

- 12 On the grid below, the distance between each dot is 1 inch.



- What is the length, in inches, of the hypotenuse of the right triangle?
- A. 4
 - B. 4.5
 - C. 5
 - D. 5.5

Reporting Category for Item 12: *Geometry*

Mathematics, Grade 8

- 13 Marisa saved \$500 to spend on a vacation. She will spend about \$45 per day on her vacation, and she must have \$70 left to pay for her bus ride home. Which of the following inequalities **best** represents the possible numbers of days, d , Marisa could be on vacation?

- A. $\$500 - (\$45 d) \geq \$70$
- B. $\$500 - (\$45 d) \leq \$70$
- C. $\$500 - (\$70 d) \geq \$45$
- D. $\$500 - (\$70 d) \leq \$45$

Reporting Category for Item 13: *Patterns, Relations, and Algebra*

- 14 What is the value of $-2[x - 2(x - y)]$ when $x = -3$ and $y = 7$?

- A. -100
- B. -34
- C. 34
- D. 27

Reporting Category for Item 14: *Patterns, Relations, and Algebra*

- 15 A bag contains 3 blue, 4 red, and 2 white marbles. Karin is going to draw out a marble without looking in the bag. What is the probability that she will **not** draw a red marble?

- A. $\frac{1}{3}$
- B. $\frac{5}{9}$
- C. $\frac{2}{3}$
- D. $\frac{4}{9}$

Reporting Category for Item 15: *Data Analysis, Statistics, and Probability*

Mathematics, Grade 8

- 16 Compute:

$$10 - (3)^2 + (-3) \times 2$$

- A. -5
- B. 4
- C. -2
- D. 2

Reporting Category for Item 16: *Number Sense and Operations*

- 17 Latoya and Keith dropped a ball from various heights and measured the height of the first bounce. They recorded their data in the chart below.

Height from which ball was dropped (d)	40 in.	60 in.	50 in.	20 in.
Height of first bounce (b)	19 in.	30 in.	27 in.	11 in.

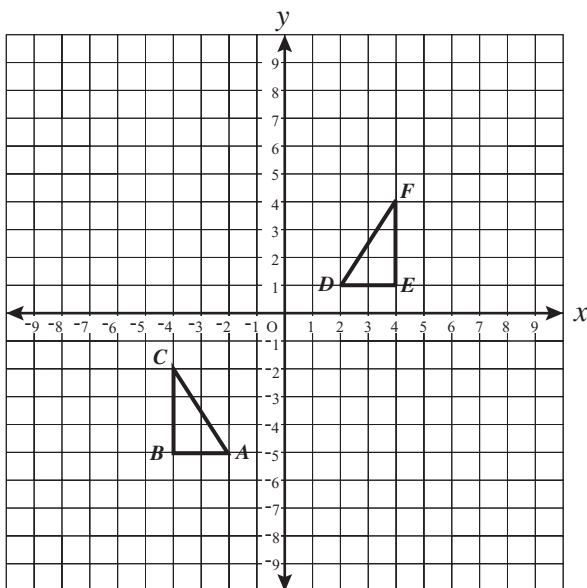
Which equation **best** shows the relationship between the height from which the ball was dropped and the height of the ball's first bounce?

- A. $b = d - 20$
- B. $b = 2d$
- C. $b = d + 20$
- D. $b = \frac{1}{2}d$

Reporting Category for Item 17: *Patterns, Relations, and Algebra*

Mathematics, Grade 8

- 18 $\triangle ABC$ and $\triangle DEF$ are shown on the grid below.



Which of the following transformations will map $\triangle ABC$ onto $\triangle DEF$?

- A. Reflect $\triangle ABC$ over the y -axis and shift up 6 spaces.
- B. Reflect $\triangle ABC$ over the x -axis and shift up 6 spaces.
- C. Reflect $\triangle ABC$ over the y -axis and shift down 6 spaces.
- D. Reflect $\triangle ABC$ over the y -axis, reflect over the x -axis, and shift down 4 spaces.

Reporting Category for Item 18: **Geometry**

Mathematics, Grade 8

Session 1, Short-Answer Questions



- 19 What is the ratio of the circumference of a circle to its diameter?

*Reporting Category for Item 19: **Number Sense and Operations***

- 20 Let x be a positive, even number that is **less than** 10. Write one ordered pair (x, y) that would make the equation $y = x + 3$ true.

*Reporting Category for Item 20: **Patterns, Relations, and Algebra***

- 21 The Eliot School plans to have the school library carpeted. The room is in the shape of a rectangle and measures 24 feet by 18 feet. If the carpet costs \$25.75 per square yard including installation, how much will it cost to have the library carpeted?

*Reporting Category for Item 21: **Measurement***

Session 1, Open-Response Question



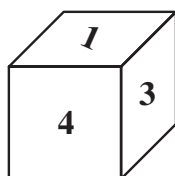
- 22** Lionel and Tracy are playing a game using two six-sided number cubes. The faces of each cube are numbered as shown below.



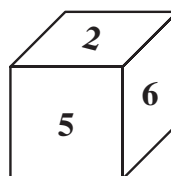
Lionel has a red cube and Tracy has a green cube. To play the game they both roll their cubes at the same time.

- The numbers that show face up when the cubes stop rolling are used to make a fraction.
- The number on the red cube is used for the numerator and the number on the green cube is used for the denominator.

For example, the results shown below would make the fraction $\frac{1}{2}$.



red
cube



green
cube

- Lionel wins 1 point if the fraction formed has a value less than one.
 - Tracy wins 1 point if the fraction has a value greater than one.
 - No one gets a point if the fraction is equal to one.
- Make a list or a table in your Student Answer Booklet of all of the fractions possible from rolling 1 red and 1 green cube. How many total different fractions are there?
 - If Lionel (red cube) rolls a 3, what is the probability that Tracy (green cube) wins 1 point? Show your work or explain how you obtained your answer.
 - Using your table, what is the probability of each player winning a point on a given turn? Do you think this game is fair to both players? Show your work or explain how you obtained your answer.

Reporting Category for Item 22: Data Analysis, Statistics, and Probability

Mathematics, Grade 8

Session 2, Multiple-Choice Questions



- 23 The table below shows the annual salaries of employees of a company based on years of employment.

Annual Salary

Years of Employment	Annual Salary
Starting Salary	\$30,000
1	\$31,500
2	\$33,000
3	\$34,500
4	\$36,000

Based on the data in the table, what is the annual salary of an employee who has just completed 10 years of service with this company?

- A. \$46,500
- B. \$45,000
- C. \$43,500
- D. \$40,000

Reporting Category for Item 23: Patterns, Relations, and Algebra

Mathematics, Grade 8

- 24 Which of the following fractions is equivalent to 0.2×0.6 ?

A. $\frac{3}{25}$

B. $\frac{12}{25}$

C. $\frac{3}{5}$

D. $\frac{6}{5}$

Reporting Category for Item 24: *Number Sense and Operations*

- 25 Mr. Gonzales is planning to drive 135 miles from West Stockbridge to Boston on the Mass Pike. He estimates that he will average 55 miles per hour. What is the latest time he can leave West Stockbridge to arrive in Boston at 11:00 A.M.?

A. 9:30 A.M.

B. 8:30 A.M.

C. 7:30 A.M.

D. 6:30 A.M.

Reporting Category for Item 25: *Measurement*

- 26 Ms. Jordan bought a box of 32 granola bars. Every day each of her three children ate one granola bar for lunch. Now there are only 11 bars left. Which equation can be used to find the number of days, n , that the children ate the bars for lunch?

A. $32 = \frac{n}{3} - 11$

B. $32 = 3n - 11$

C. $32 = \frac{n}{3} + 11$

D. $32 = 3n + 11$

Reporting Category for Item 26: *Patterns, Relations, and Algebra*

Mathematics, Grade 8

Use the equation below to answer question 27.

$$-3xy = 45$$

- 27 Which of the following statements is true?
- A. Only x is negative.
 - B. Only y is negative.
 - C. Both x and y are negative.
 - D. Either x or y is negative.

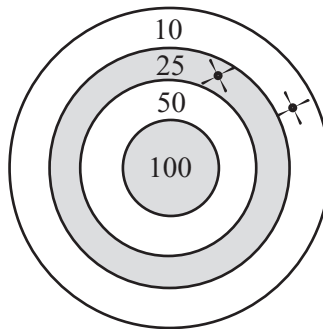
Reporting Category for Item 27: **Number Sense and Operations**

Mathematics, Grade 8

Session 2, Open-Response Questions



- 28 Esther shot two arrows at a target. The picture below shows where the arrows landed.



Esther calculated her score by adding the number of points for each ring in which an arrow landed. For the two arrows above, her score was 35 points ($25 + 10$).

- In your Student Answer Booklet, make a list of all the possible scores Esther could have gotten by shooting two arrows that hit the target.
- Is it possible for Esther to score a total of 235 points using **only** 5 arrows? Show your work or explain your answer.
- What is the **fewest** number of arrows required for Esther to score a total of 240 points? Show your work or explain your answer.

Reporting Category for Item 28: Number Sense and Operations

Mathematics, Grade 8

- 29 Molly formed three polygons—a triangle, a rectangle, and a pentagon—with string. She calculated the sum of the measures of the interior angles for each polygon and entered her data in the chart shown below.

Type of Polygons	Number of Sides	Sum of the Measures of the Interior Angles
Triangle	3	180°
Rectangle	4	360°
Pentagon	5	540°
Hexagon	6	?
Octagon	8	?
Unnamed Polygon	?	2340°
n -sided Polygon	n	?

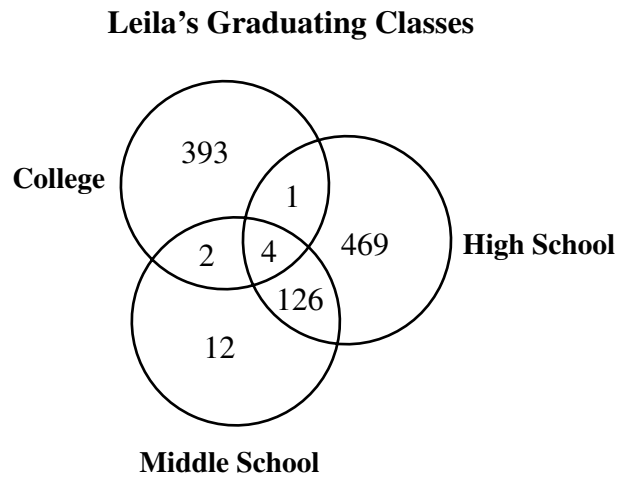
- What is the sum of the measures of the interior angles of a hexagon?
- What is the sum of the measures of the interior angles of an octagon?
- How many sides does an unnamed polygon have if the sum of the measures of the interior angles is 2340°?
- Explain how you would find the sum of the measures of the interior angles of an n -sided polygon.

Reporting Category for Item 29: **Geometry**

Session 2, Multiple-Choice Questions



- 30 The Venn diagram below shows Leila’s graduating classes from middle school, high school, and college.



How many students graduated together from **both** Leila’s middle school and high school?

- A. 133
- B. 132
- C. 131
- D. 130

Reporting Category for Item 30: *Data Analysis, Statistics, and Probability*

Mathematics, Grade 8

- 31 The chart shows the area of the eight largest counties in Massachusetts.

**Area of Selected
Massachusetts Counties**

Selected Counties in Massachusetts	Area (square miles)
Berkshire	931
Bristol	556
Franklin	702
Hampden	618
Hampshire	529
Middlesex	824
Plymouth	661
Worcester	1513

What is the median area, to the nearest square mile, of the 8 largest counties in Massachusetts?

- A. 661 square miles
- B. 682 square miles
- C. 702 square miles
- D. 792 square miles

Reporting Category for Item 31: Data Analysis, Statistics, and Probability

Mathematics, Grade 8

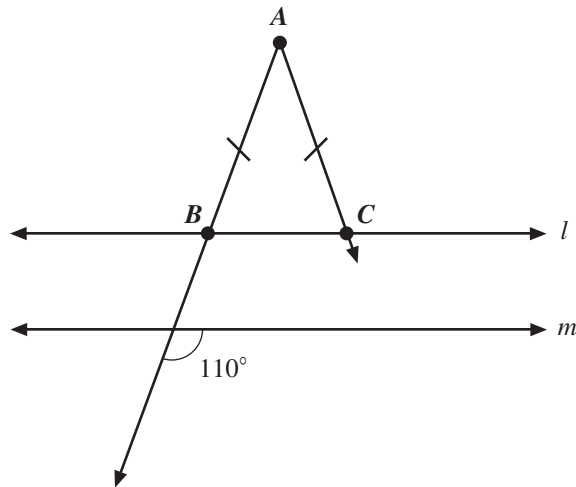
- 32 What value of x makes the equation below true?

$$3x + 2(x - 5) = 50$$

- A. 8
- B. 9
- C. 11
- D. 12

Reporting Category for Item 32: *Patterns, Relations, and Algebra*

Use the figure below to answer question 33.



- 33 In the figure shown above, lines l and m are parallel, and $\triangle ABC$ is isosceles. What is the measure of $\angle ABC$?

- A. 40°
- B. 50°
- C. 60°
- D. 70°

Reporting Category for Item 33: *Geometry*

Mathematics, Grade 8

- 34 Which equation states a rule for the pattern shown in the table below?

Input (x)	1	2	3	4
Output (y)	1	5	11	19

- A. $y = x^2 - x + 1$
- B. $y = x^2 + x - 1$
- C. $y = x^2 + 3$
- D. $y = x^2 + 1$

Reporting Category for Item 34: *Patterns, Relations, and Algebra*

- 35 In the equation shown below, x represents a positive real number.

$$y = \frac{100}{x} + 50$$

As the value of x gets larger, what happens to the value of y ?

- A. The value of y stays the same.
- B. The value of y increases.
- C. The value of y approaches 50.
- D. The value of y approaches 100.

Reporting Category for Item 35: *Patterns, Relations, and Algebra*

- 36 Which is equivalent to $p \div \frac{1}{10}$?

- A. $10p$
- B. $0.1p$
- C. $\frac{p}{10}$
- D. $0.01p$

Reporting Category for Item 36: *Number Sense and Operations*

Mathematics, Grade 8

- 37 The chart below shows the amount spent by customers at a department store on a typical business day.

Amount Spent	Number of Customers
\$0	158
\$0.01 - \$5.99	94
\$6.00 - \$9.99	203
\$10.00 - \$19.99	126
\$20.00 - \$49.99	47
\$50.00 - \$99.99	38
\$100 and over	53

Based on the information in the chart, which of the following is closest to the probability that a customer entering the store on a typical day will spend **at least** \$10?

- A. 13%
- B. 18%
- C. 37%
- D. 81%

Reporting Category for Item 37: *Data Analysis, Statistics, and Probability*

- 38 The computer game Peter wants to buy will cost at least \$50 and not more than \$70. He earns \$3 an hour running errands for his grandmother. Which inequality shows the number of hours, n , he will have to work to pay for the game?

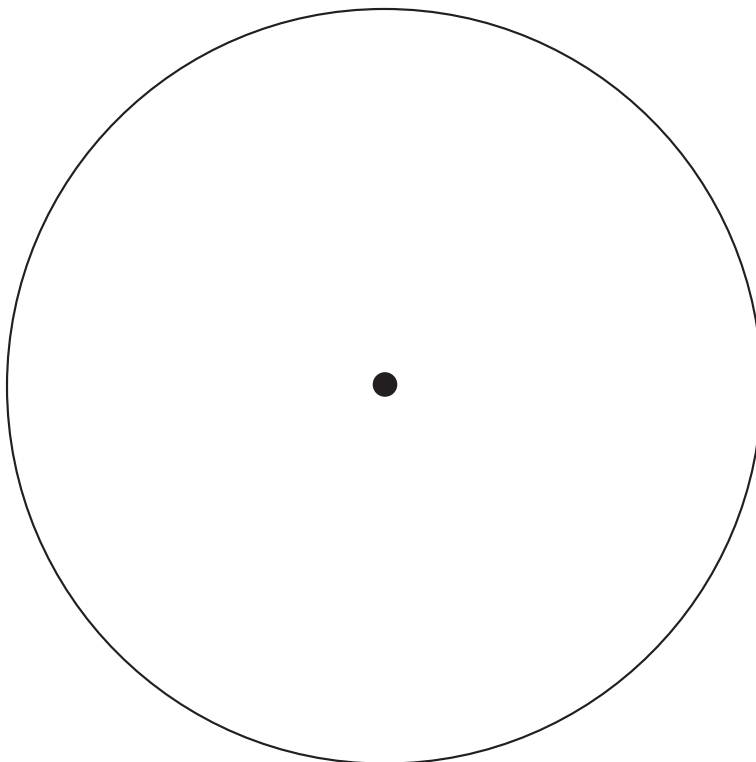
- A. $3n \geq 20$
- B. $\frac{n}{3} \geq 20$
- C. $50 \leq 3n \leq 70$
- D. $50 \leq \frac{n}{3} \leq 70$

Reporting Category for Item 38: *Patterns, Relations, and Algebra*

Session 2, Open-Response Question



Use the ruler included in your reference sheet to answer question 39.



- 39 The figure shown above represents the base of a cylindrical tank. The tank has a height of 16 centimeters (1 milliliter = 1 cubic centimeter).
- What is the radius of the base, in centimeters?
 - What is the volume of the cylinder in milliliters? Show your work.
 - If both the radius and the height of the cylinder were doubled, what would be the volume of the cylinder in milliliters? Show your work.
 - Based on your answers to parts b and c, what is the ratio of the volume of the smaller tank to the volume of the larger tank? Show your work.

Reporting Category for Item 39: *Measurement*